MPhil ACS Research Essay Proposal:
Type Theory for the Category of Containers

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Summary

Containers are a mathematical representation of algebraic datatypes. A container is a collection of shapes and, for each shape, a collection of positions. A particular instance of a datatype is given by a choice of shape and an assignment of data to the positions of that shape. A container also represents an endofunctor on the category of sets, taking a set of data to the set of all datatypes (of particular shapes) over that data. In fact, containers form a full subcategory of the category of endofunctors [1]. The category of containers has been shown to be closed under many categorical constructions, including products, coproducts, initial algebras, terminal coalgebras, and power objects [2].

Closure of a category under such constructions corresponds to properties of the internal logic and type theory associated with that category. This relationship is one of the themes of the field known as categorical logic [3]. Because of their interpretation as datatypes, containers are more concrete and intuitive than arbitrary endofunctors. My proposal is to investigate the structure of the category of containers from the categorical logic perspective, looking especially at how its type theory is related to that of the ambient category of all endofunctors. The ultimate motivation for investigations along these lines is a better understanding of algebraic datatypes for programming and language design.

Plan

08/3 − 20/3 Collect and digest a bibliography of papers about containers.

21/3 − 23/3 List all existing results about the category of containers.

24/3 − 14/4 Write (or collate where applicable) detailed proofs of the existing results.

15/4 − 14/5 Work on open questions about the category of containers.

15/5 − 31/5 Catch up if behind, do further investigation, or mechanise the proofs.

01/6 − 14/6 Write the essay.

15/6 − 17/6 Do final checks.

References

[1] Michael Abbott, Thorsten Altenkirch, and Neil Ghani. Categories of containers. In Andrew D. Gordon, editor, *FoSSaCS*, volume 2620 of *Lecture Notes in Computer Science*, pages 23–38. Springer, 2003.

[2] Thorsten Altenkirch, Paul Levy, and Sam Staton. Higher-order containers. In Fernando Ferreira, Benedikt Löwe, Elvira Mayordomo, and Luís Mendes Gomes, editors, *CiE*, volume 6158 of *Lecture Notes in Computer Science*, pages 11–20. Springer, 2010.

[3] A. M. Pitts. Categorical logic. In S. Abramsky, D. M. Gabbay, and T. S. E. Maibaum, editors, *Handbook of Logic in Computer Science, Volume 5. Algebraic and Logical Structures*, pages 39–128. Oxford University Press, 2000.